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THE QUANTUM GEOMETRY OF $N = (2, 2)$ NON-LINEAR σ-MODELS

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ABSTRACT

We consider a general $N = (2, 2)$ non-linear σ-model in $(2, 2)$ superspace. Depending on the details of the complex structures involved, an off-shell description can be given in terms of chiral, twisted chiral and semi-chiral superfields. Using superspace techniques, we derive the conditions the potential has to satisfy in order to be ultra-violet finite at one loop. We pay particular attention to the effects due to the presence of semi-chiral superfields. A complete description of $N = (2, 2)$ strings follows from this.

Ever since Zumino's observation [1] that the target manifold of a four-dimensional supersymmetric σ-model needs to be Kähler, it has been recognized that supersymmetry and complex geometry go hand in hand. In two dimensions this relation becomes particularly rich. In this paper, we study non-linear σ-models with $N = (2, 2)$ supersymmetry. Such models are not only the building blocks for type II superstrings, but they describe the matter sector of $N = (2, 2)$ string theories as well. Classically, $(2, 2)$ supersymmetry requires the existence of two covariantly constant complex structures, which are such that the metric is hermitean for both. The simplest choice, both complex structures equal, implies that the target manifold is Kähler. The other cases generalize the notion of Kähler geometry in the sense that the geometry can always locally be described in terms of a single potential. For such models the ultra-violet divergences are much milder than what can be expected from power counting. However, requiring ultra-violet finiteness imposes further restrictions on this potential. Up till now, this has only been studied for target manifolds where the supersymmetry algebra closes off-shell in $(1, 1)$ superspace. This is equivalent to the requirement that both complex structures commute. In the present paper, we examine the generic case, where off-shell closure requires a $(2, 2)$ superspace formulation.

[1] Aspirant NFWO

An arbitrary $N=(1,1)$ supersymmetric non-linear σ-model has an action in $(1,1)$ superspace [2] given by[2]

$$\mathcal{S}=\frac{1}{2\pi}\int d^2xd^2\tilde{\theta}\,(g_{ab}+b_{ab})\,\tilde{D}_+\phi^a\tilde{D}_-\phi^b, \tag{1}$$

where the metric on the target manifold is g_{ab} while $b_{ab}=-b_{ba}$ is the torsion potential:

$$T^a{}_{bc}\equiv-\frac{3}{2}g^{ad}b_{[bc,d]}. \tag{2}$$

This model is $N=(2,2)$ supersymmetric, provided there exists two complex structures J_+ and J_-, which are covariantly constant,

$$\nabla^+_c J^a_{+b}=\nabla^-_c J^a_{-b}=0, \tag{3}$$

where $\nabla^\pm$ denotes covariant differentiation using the $\Gamma_\pm$ connections:

$$\Gamma^a_{\pm bc}\equiv\left\{{}^{\,a}_{bc}\right\}\pm T^a{}_{bc}, \tag{4}$$

the first term being the standard Levi-Cevita connection. Finally, the metric should be hermitean w.r.t. both complex structures. For a four-dimensional target manifold this simplifies as one can show that any covariantly constant *almost* complex structure for which the metric is hermitean is automatically a complex structure [3], i.e. the Nijenhuis tensor vanishes trivially. On-shell, one gets the standard $N=(2,2)$ supersymmetry algebra, while the off-shell non-closure terms are proportional to the commutator $[J,\bar{J}]$.

Building on the results of [4], it was argued in [5] that, in order to achieve a manifest $(2,2)$ supersymmetric description of these models, *i.e.* a formulation in $(2,2)$ superspace, chiral, twisted chiral [2] and semi-chiral [6] fields are sufficient. More explicitly: the tangent space at any point of the target manifold can be decomposed into three subspaces: $\ker(J_+-J_-)\oplus\ker(J_++J_-)\oplus(\ker[J_+,J_-])^\perp$. These subspaces are conjectured to be integrable to chiral, twisted chiral and semi-chiral coordinates resp. In $N=(2,2)$ superspace, we have four fermionic coordinates, denoted by θ^+, $\bar{\theta}^+$, θ^- and $\bar{\theta}^-$ with covariant derivatives D_+, $\bar{D}_+$, D_- and $\bar{D}_-$. The only non-vanishing anticommutators are: $\{D_+,\bar{D}_+\}=\partial_{\ddagger}$ and $\{D_-,\bar{D}_-\}=\partial_{=}$. Note that the $N=(1,1)$ fermionic coordinates previously introduced are given by the real parts of the $N=(2,2)$ coordinates, $\tilde{\theta}^\pm=(\theta^\pm+\bar{\theta}^\pm)/2$. The three types of superfields are defined through the following constraints.

i. <u>Chiral superfields</u>: z^a, $\bar{z}^{\bar{a}}$; a, $\bar{a}\in\{1,\cdots d_c\}$.
$D_\pm z^a=\bar{D}_\pm\bar{z}^{\bar{a}}=0$.

[2]We take $\tilde{D}_+\equiv\frac{\partial}{\partial\tilde{\theta}^+}+\tilde{\theta}^+\partial_{\ddagger}$ and $\tilde{D}_-\equiv\frac{\partial}{\partial\tilde{\theta}^-}+\tilde{\theta}^-\partial_{=}$. Our two-dimensional metric is such that its only nonvanishing component is $g_{\ddagger=}=1$.

ii. Twisted chiral superfields: w^m, $\bar{w}^{\bar{m}}$; $m, \bar{m} \in \{1, \cdots d_t\}$
$D_+ w^m = \bar{D}_- w^m = \bar{D}_+ \bar{w}^{\bar{m}} = D_- \bar{w}^{\bar{m}} = 0.$
iii. Semi-chiral superfields[3]: r^α, $\bar{r}^{\bar{\alpha}}$, $s^{\tilde{\alpha}}$, $\bar{s}^{\tilde{\bar{\alpha}}}$; $\alpha, \bar{\alpha}, \tilde{\alpha}, \tilde{\bar{\alpha}} \in \{1, \cdots, d_s\}$.
$D_+ r^\alpha = \bar{D}_+ \bar{r}^{\bar{\alpha}} = \bar{D}_- s^{\tilde{\alpha}} = D_- \bar{s}^{\tilde{\bar{\alpha}}} = 0.$

The constraints reduce the number of components of chiral and twisted chiral superfields to those of an $N = (1,1)$ superfield. Semi-chiral superfields have twice as many components, half of which are auxiliary. This is not so surprising as chiral and twisted chiral superfields parametrize $\ker[J_+, J_-] = \ker(J_+ - J_-) \oplus \ker(J_+ + J_-)$ where the $N = (2,2)$ algebra closes off-shell.

The action in $(2,2)$ superspace is

$$\mathcal{S} = \int d^2x d^4\theta K(z, \bar{z}, w, \bar{w}, r, \bar{r}, s, \bar{s}), \tag{5}$$

with K a real potential. The potential is determined modulo a generalized Kähler transformation:

$$K \simeq K + f(z, w, r) + \bar{f}(\bar{z}, \bar{w}, \bar{r}) + g(z, \bar{w}, \bar{s}) + \bar{g}(\bar{z}, w, s). \tag{6}$$

Starting from eq. (5), one can pass to $N = (1,1)$ superspace. Upon elimination of the auxiliary fields, one gets, by comparing the result to eq. (1) and the supersymmetry transformation rules, explicit expressions for the metric, torsion and complex structures in terms of the potential [5]. Various explicit examples are known. Kähler manifolds are described using chiral fields only. The $SU(2) \times U(1)$ Wess-Zumino-Witten (WZW) model can be described either in terms of a chiral and a chiral multiplet [7] or in terms of one semi-chiral multiplet [5, 4]. This ambiguity reflects the freedom one has in choosing the left and right complex structures. The WZW model on $SU(2) \times SU(2)$ is described in terms of a semi-chiral and a twisted chiral field [5]. Even hyper-Kähler manifolds can be described in terms of semi-chiral coordinates [5]. Indeed, choosing J_+ and J_- such that $\{J_+, J_-\} = 0$, one gets $\ker[J_+, J_-] = \emptyset$.

Conformal invariance of these models puts strong restrictions on the allowed potentials K in eq. (5). The requirement that the one loop β-function vanishes [8],

$$R^+_{ab} + 2\nabla^-_a \partial_b \Phi = 0, \tag{7}$$

with Φ the dilaton, yields various dilaton configurations together with differential equations for the potential[4].

[3]Our notation is slightly misleading. While J_+ and J_- are both diagonal for chiral and twisted chiral superfields, this is not so for semi-chiral superfields. This is evident from the fact that semi-chiral fields parametrize $(\ker[J_+, J_-])^\perp$.

[4]Our convention for the curvature tensor is $R^a{}_{bcd} = \Gamma^a{}_{bd,c} + \Gamma^a{}_{ec}\Gamma^e{}_{bd} - (c \leftrightarrow d)$. Pending upon whether we use the Γ_+ or the Γ_- connection, we get two curvature tensors related by $R^+_{abcd} = R^-_{cdab}$. The Ricci tensors $R^\pm_{ab}$ are defined by $R^\pm_{ab} \equiv R^{\pm c}{}_{acb}$ and $R^+_{ab} = R^-_{ba}$. Covariant derivatives are taken as $\nabla^+_b V^a \equiv V^a{}_{,b} + \Gamma^{+a}{}_{cb} V^c$ and $\nabla^+_b V_a \equiv V^a{}_{,b} - \Gamma^{+c}{}_{ab} V^c$.

So our task looks simple: starting from eq. (5) we pass to $N=(1,1)$ superspace, eliminate the auxiliary fields, and compare with eq. (1). In that way, we get explicit expressions for metric and torsion in terms of the potential, to use in the β-function eq. (7) which can then be analysed. This is indeed straightforward as long as only chiral and twisted chiral fields are present. Having only chiral fields and choosing the dilaton constant, yields [9]

$$\det K_{a\bar{b}} = 1, \tag{8}$$

where $K_{a\bar{b}}$ stands for $\partial^2 K/\partial z^a \partial z^{\bar{b}}$. When both chiral and twisted chiral superfields are present and the dilaton is chosen as $\Phi = (1/2)\ln\det K_{a\bar{b}}$, eq. (7) is equivalent to [10]

$$\det K_{a\bar{b}} = \det(-K_{m\bar{n}}), \tag{9}$$

More general solutions, where the dilaton is a potential for a holomorphic Killing vector, were studied in [11]. Once semi-chiral fields enter the game, the metric and torsion (see e.g. [5] or [6]) become so complicated that the whole program becomes technically unfeasable. The way out of course, is to recompute the β-functions, but now directly in $(2,2)$ superspace. At present, there is no good understanding of the dilaton in the presence of semi-chiral sperfields, so in this paper we limit ourselves to the study of a necessary condition for conformal invariance: ultra-violet finiteness. In fact, ultra-violet finiteness for chiral fields is precisely equivalent to eq. (8), while for chiral and twisted chiral fields it is equal to eq. (9).

Some care is required in setting up the Feynman rules. As an example, we compute the one loop UV divergence for a potential which depends on one semi-chiral multiplet. The only subtle part is the construction of the free field propagators. A particularly convenient potential is given by $K_0 = 1/2(r\bar{r}+s\bar{s}) + r\bar{s} + s\bar{r}$. We introduce sources, j, $\bar{j}$, k and $\bar{k}$, which are unconstrained superfields. The complete action can be rewritten as

$$\begin{aligned}
\mathcal{S} &= \int d^2x d^4\theta \left(K_0 + jr + \bar{j}\bar{r} + ks + \bar{k}\bar{s}\right) \\
&= \int d^2x d^4\theta \left(\frac{1}{2}(r\bar{r}+s\bar{s}) + \frac{D_+\bar{D}_+}{\partial_{\ddagger}} r \frac{D_-\bar{D}_-}{\partial_{=}}\bar{s} + \frac{\bar{D}_-D_-}{\partial_{=}} s \frac{\bar{D}_+D_+}{\partial_{\ddagger}}\bar{r} + \right. \\
&\quad \left. r\frac{\bar{D}_+D_+}{\partial_{\ddagger}} j + \bar{r}\frac{D_+\bar{D}_+}{\partial_{\ddagger}}\bar{j} + s\frac{D_-\bar{D}_-}{\partial_{=}} k + \bar{s}\frac{\bar{D}_-D_-}{\partial_{=}}\bar{k}\right) \\
&= \int d^2x d^4\theta \left(K_0(\hat{r},\hat{\bar{r}},\hat{s},\hat{\bar{s}}) + \frac{4}{3}\bar{j}\frac{\bar{D}_+D_+D_-\bar{D}_-}{\partial_{\ddagger}\partial_{=}} k + \frac{4}{3}\bar{k}\frac{\bar{D}_+D_+D_-\bar{D}_-}{\partial_{\ddagger}\partial_{=}} j + \right. \\
&\quad \left. \bar{j}\frac{\bar{D}_+D_+}{\partial_{\ddagger}}\left(2 - \frac{8}{3}\frac{D_-\bar{D}_-}{\partial_{=}}\right) j + \bar{k}\frac{D_-\bar{D}_-}{\partial_{=}}\left(2 - \frac{8}{3}\frac{\bar{D}_+D_+}{\partial_{\ddagger}}\right) k\right).
\end{aligned} \tag{10}$$

The slightly unconventional rewriting of K_0 in the second line which uses the semi-chiral properties, is needed in order that the shifted fields in the standard Gaussian integral,

$$\begin{aligned}\hat{r} &\equiv r+\frac{D_+\bar{D}_+}{\partial_{\ddagger}}(2-\frac{8}{3}\frac{\bar{D}_-D_-}{\partial_=})j+\frac{4}{3}\frac{D_+\bar{D}_+\bar{D}_-D_-}{\partial_{\ddagger}\partial_=}k,\\ \hat{s} &\equiv s+\frac{\bar{D}_-D_-}{\partial_=}(2-\frac{8}{3}\frac{D_+\bar{D}_+}{\partial_{\ddagger}})k+\frac{4}{3}\frac{D_+\bar{D}_+\bar{D}_-D_-}{\partial_{\ddagger}\partial_=}j,\end{aligned} \tag{11}$$

satisfy the same constraints as r and s. From eq. (10), we immediately obtain the free field propagators.

Next, we consider a general potential $K(r',\bar{r}',s',\bar{s}')$. In order to perform the one-loop calculation, we make a linear background-quantum splitting: $r'=r_c+r$ and $s'=s_c+s$, where r_c and s_c are fixed background configurations and r and s describe the quantum fluctuations around this background. Expanding the potential around the background yields the interaction terms. As usual, the terms linear in the quantum fields are irrelevant, while for the one-loop computation, only the quadratic terms contribute:

$$\begin{aligned}\mathcal{S}_{int} &= \int d^2x d^4\theta \Big((K_{r\bar{r}}-\frac{1}{2})r\bar{r}+(K_{s\bar{s}}-\frac{1}{2})s\bar{s}+(K_{r\bar{s}}-1)r\bar{s}+(K_{s\bar{r}}-1)s\bar{r}+\\ &\frac{1}{2}K_{rr}\,rr+\frac{1}{2}K_{\bar{r}\bar{r}}\,\bar{r}\bar{r}+\frac{1}{2}K_{ss}\,ss+\frac{1}{2}K_{\bar{s}\bar{s}}\,\bar{s}\bar{s}+\text{ 3rd order terms}\Big).\end{aligned} \tag{12}$$

The D-algebra is greatly simplified by the observation in [12]: the background dependent terms can be treated as constants. Indeed, as the superspace measure is dimensionless, the counterterms are necessarily dimensionless themselves. Therefore, any derivative hitting the background terms cannot give rise to UV divergent contributions. A similar dimensional argument implies that the last four interaction terms in eq. (12) do not contribute to the UV divergence as the D algebra always gives rise to derivatives acting on the background dependent terms. Furthermore, one argues along exactly the same lines as in [12], that non-covariant contributions to the counterterms will be absent. The one-loop computation is now completely standard (using the techniques described in e.g. [13]) and leads to the counterterm:

$$\mathcal{S}^{(1)}=\frac{1}{2\pi\varepsilon}\int d^2x d^4\theta \ln\frac{1}{3}\frac{K_{r\bar{s}}K_{s\bar{r}}-K_{r\bar{r}}K_{s\bar{s}}}{K_{r\bar{r}}K_{s\bar{s}}-K_{rs}K_{\bar{r}\bar{s}}}. \tag{13}$$

From this we get the condition for UV finiteness at one loop:

$$\frac{K_{r\bar{s}}K_{s\bar{r}}-K_{r\bar{r}}K_{s\bar{s}}}{K_{r\bar{r}}K_{s\bar{s}}-K_{rs}K_{\bar{r}\bar{s}}}=\pm|F(r)|^2|G(s)|^2, \tag{14}$$

where F and G are arbitrary functions of r and s resp. Note that there is no coordinate transformation compatible with the constraints, which can remove $|F(r)|^2|G(s)|^2$.

The $+$ and $-$ signs hold for manifolds with $(4,0)$ and $(2,2)$ signature[5]. This result can immediately be verified using some examples which are known to be UV finite. The WZW model on $SU(2)\times U(1)$ is described by a semi-chiral multiplet [5, 4] with potential

$$K=-r\bar{r}+\bar{r}\bar{s}+rs-2i\int^{\bar{s}-s}dx\,\ln(1+\exp\frac{i}{2}x), \tag{15}$$

and we find $F=1$ and $G=\exp(-is/2)$. Another class of interesting examples are the 4-dimensional hyper-Kähler manifolds where J_+ and J_- are choosen to be anti-commuting. The potential satisfies then [5] $|K_{rs}|^2+|K_{r\bar{s}}|^2=2K_{s\bar{s}}K_{r\bar{r}}$ and $F=G=1$.

The computation can immediately be generalized to the case where d_s semi-chiral multiplets are present. The one-loop counterterm reads then

$$\mathcal{S}^{(1)}=\frac{1}{2\pi\varepsilon}\int d^2xd^4\theta\,\ln\frac{\det\mathcal{N}_2}{\det(\sqrt{-3}\mathcal{N}_1)}, \tag{16}$$

where

$$\mathcal{N}_1\equiv\begin{pmatrix}K_{\tilde{\alpha}\beta} & K_{\tilde{\alpha}\tilde{\bar{\beta}}}\\ K_{\bar{\alpha}\beta} & K_{\bar{\alpha}\tilde{\bar{\beta}}}\end{pmatrix},\qquad \mathcal{N}_2\equiv\begin{pmatrix}K_{\tilde{\alpha}\bar{\beta}} & K_{\tilde{\alpha}\tilde{\bar{\beta}}}\\ K_{\alpha\bar{\beta}} & K_{\alpha\tilde{\bar{\beta}}}\end{pmatrix}. \tag{17}$$

In [5], it was shown that non-degeneracy of the target manifold metric is equivalent to $\det\mathcal{N}_1\neq 0$ and $\det\mathcal{N}_2\neq 0$.

Finally, one can also repeat the calculation for a general potential, eq. (5), which depends on all three types of superfields. One finds the counterterm

$$\mathcal{S}^{(1)}=\frac{1}{2\pi\varepsilon}\int d^2xd^4\theta\,\ln\frac{\det(-K_{m\bar{n}})\det\mathcal{N}_2'}{\det K_{a\bar{b}}\det(\sqrt{-3}\mathcal{N}_1')}, \tag{18}$$

where $\mathcal{N}_1'$ and $\mathcal{N}_2'$ are similar to $\mathcal{N}_1$ and $\mathcal{N}_2$ in eq. (17) except that in $\mathcal{N}_1'$ one has to write the $d_s\times d_s$ matrices $(K_{AB})-(K_{Aa})(K_{\bar{a}b})^{-1}(K_{\bar{b}B})$ instead of the matrices K_{AB}, while in $\mathcal{N}_2'$ one has $(K_{AB})-(K_{Am})(K_{\bar{m}n})^{-1}(K_{\bar{n}B})$ instead of K_{AB}. In these expressions, the capital letters denote the indices appearing in eq. (17), while the small indices denote derivatives w.r.t. chiral or twisted chiral fields, following the notation introduced in the definition of these fields. Again, this result can be verified using a non-trivial example. In [5], $SU(2)\times SU(2)$ was described in terms of a semi-chiral multiplet r, s and a chiral field z with potential

$$\begin{aligned}K &= -z\bar{z}+z\bar{r}+\bar{z}r+isz-i\bar{s}\bar{z}+i\bar{s}\bar{r}-isr\\ &\quad -i\int^{\bar{r}-r}dy\,\ln(1-\exp iy)-i\int^{\bar{s}-s}dy\,\ln(1-\exp iy).\end{aligned} \tag{19}$$

[5]Though our computation was done for $(4,0)$ signature, it can easily be redone for $(2,2)$ signature. One just replaces the quadratic potential by $K_0=1/2(r\bar{r}-s\bar{s})+r\bar{s}+s\bar{r}$ and one gets the same counterterm as in eq. (13), except that the factor 3 in the denominator is replaced by a factor -5.

Using the previous expression and taking eq. (6) into account, one easily verifies that this potential is indeed finite at one loop.

From the results in [14], one expects new counterterms to appear from four loops on. Using arguments similar to those in [15] one can presumably argue that a deformation of the potential can be defined order by order so that they vanish.

Finally we consider the four dimensional case which is relevant for the study of $N = (2,2)$ strings. There are three possible backgrounds: those given in terms of two chiral fields, those described by one chiral and one twisted chiral and finally those consisting of a single semi-chiral multiplet. Note that having two twisted chiral fields only is isomorphic to having only two chiral fields.

In [16], $N = (2,2)$ strings in a Kähler background were studied[6]. A description in terms of two chiral fields can be given and ultra-violet finiteness requires that the potential satisfies the Monge-Ampère equation:

$$K_{z_1\bar{z}_1}K_{z_2\bar{z}_2} - K_{z_1\bar{z}_2}K_{z_2\bar{z}_1} = \pm 1, \tag{20}$$

where the $+$ and $-$ sign stands for a $(4,0)$ and $(2,2)$ signature of space-time resp. This can be integrated to the Plebanski action

$$\mathcal{S}_{eff} = \int d^2z_1 d^2z_2\, K \left\{ \frac{1}{3}\left(K_{z_1\bar{z}_1}K_{z_2\bar{z}_2} - K_{z_1\bar{z}_2}K_{z_2\bar{z}_1}\right) \mp 1 \right\}. \tag{21}$$

In [17, 10, 11], backgrounds consisting of one chiral and one twisted chiral field were studied. UV finiteness is equivalent to the Laplace equation

$$K_{z\bar{z}} \pm K_{w\bar{w}} = 0, \tag{22}$$

where again the $+$ and $-$ signs correspond to $(4,0)$ or $(2,2)$ signature resp. In this case the $N = 2$ string describes a free scalar field, with effective action:

$$\mathcal{S}_{eff} = \frac{1}{2}\int d^2z d^2w \left\{ K_z K_{\bar{z}} \pm K_w K_{\bar{w}} \right\}. \tag{23}$$

Finally, having a background consisting of a semi-chiral multiplet yields eq. (14) as condition on the potential. Only when $|F(r)|^2|G(s)|^2 = a$, with a a positive real constant, can this be integrated to an action cubic in the fields,

$$\begin{aligned} \mathcal{S}_{eff} \;\; &= \;\; \frac{1}{3}\int d^2r d^2s\, K \left\{ K_{r\bar{s}}K_{r\bar{s}} \pm a K_{rs}K_{\bar{r}\bar{s}} - \right. \\ & \qquad \left. (1 \pm a)\, K_{r\bar{r}}K_{s\bar{s}} \right\}. \end{aligned} \tag{24}$$

For the former two cases, one can show that the holonomy group is contained in $SU(2)$, hence these models describe gravitational instantons. Whether or not this is also true for the latter case is being examined.

[6] In [16] a different path was taken. The effective action was obtained by directly studying the scattering amplitudes. The dynamics was given by a scalar field Φ, satisfying the Plebanski equation. This can be obtained from the Monge-Ampère equation by setting $K = K_0 + \Phi$ with $K_0 = z_1\bar{z}_1 \pm z_2\bar{z}_2$.

An important point which still has to be elucidated is the role of the dilaton. In [18], this was investigated for backgrounds consisting of chiral and twisted chiral superfields. It was found that the geometry of the $(2,2)$ super worldsheet implies the existence of four types ((anti-)chiral and twisted (anti-)chiral) of worldsheet curvatures, which couple to fields satisfying the same constraints. A generalization of this in the presence of semi-chiral superfields is presently under study.

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